Explanation of Thermal Rectification

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Abstract

IN an effort to understand the physical basis of thermal rectification in conduction, a semiquantitative study of an idealized contact between two smooth, frictionless spheres has been undertaken. Hertzian methods were employed to express the contact length as a function of the radii of the spheres, the compressive force on the spheres, and the mechanical properties of the spheres. Next, an expression defining the relationship between the contact length and the heat flux through the function was derived. The results of this analysis reveal that distortion at the contact surfaces of the two spheres due to thermal strains within the spheres can explain the reported behavior of thermal rectification.

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Thermal rectification (or directional bias) has been observed at contacts between both dissimilar and similar materials. Although experimental investigations have confirmed the existence of thermal rectification, this phenomenon continues to be enigmatic. This is due to the large number of parameters that can affect the resistance at the junction between the two materials. Contact geometry: 1) interfacial surface roughness; 2) interfacial surface waviness; 3) mean slope of the individual roughness asperities on each of the contact surfaces; 4) number, size, and shape of the asperity contacts. Loading conditions: 5) interfacial contact pressure; 6) loading history. Thermal conditions: 7) temperatures at the contact surfaces; 8) heat-flux magnitude and direction; 9) thermal history of the contact. Material properties: 10) physical properties of the contacting materials (most notably, the thermal coefficient of expansion α, thermal conductivity k, hardness, elastic modulus E, and Poisson's ratio ν; 11) physical properties of any interstitial materials.

Researchers have published conflicting observations of the influence of these parameters on the directional bias as reviewed by Somers.[1] No theoretical or empirical model has been published to date that can predict consistently accurate resistance values and rectification trends for a wide range of contact conditions. Moreover, none of the published analytic techniques explain all the seemingly contradictory experimental observations of thermal rectification.

To understand the physics underlying thermal rectification, a semiquantitative model of an idealized contact between two elastic materials was developed. The purpose of this investigation was to determine the relationship between thermal rectification and thermal strain at the interface between two contacting materials.

Presented as Paper 84-0398 at the AIAA 22nd Aerospace Sciences Meeting, Reno, NV, Jan. 9-12, 1984; received June 18, 1984; synoptic received Sept. 17, 1986. Full paper available from National Technical Information Service, Springfield, VA 22151, at the standard price (available upon request).

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Description of the Contact Model

Consider the macroscoptic contact between two elastic spheres of radii R_1 and R_2, which are loaded mechanically by a force F normal to the contact (Fig. 1). This contact geometry is not unlike the contact between the end faces of two right coaxial cylinders, as used in many experimental apparatuses. If no tangential tractions exist at the interface between the two spheres, Hertz demonstrated that the contact is circular, with a radius r_0:

$$r_0 = \{[3\pi F(K_1+K_2)R_1R_2]/4(R_1+R_2)\}^{1/3} \tag{1}$$

where

$$K_1 = (1-\nu_1^2)/\pi E_1$$

$$K_2 = (1-\nu_2^2)/\pi E_2$$

If a heat flux q flows through the contact, Eq. (1) remains valid, provided the thermomechanical properties of the two materials are not temperature dependent and R_1 and R_2 represent the final surface geometries of the materials. The contact radius may be used to compute the thermal resistance R_c of the contact, provided the magnitude of ΔT across the interface is known:

$$R_c = \Delta T/q \tag{2}$$

A change of heat-flux magnitude flowing through the interface would lead to different temperature fields in the two materials, causing thermal distortion of the contact surfaces. This would alter the contact area and consequently change the thermal resistance of the junction. So long as both surfaces remain convex, they will always be connected by a single contact area. Also, due to the assumption of a frictionless contact, the thermal and mechanical analyses of the contact can be decoupled, permitting the independent calculation of the distortion of the interfaces for a specified heat flux, followed by the computation of the contact area using Eq. (1). Therefore, if it is assumed that the surfaces of the two contacting bodies remain spherical, the relative change in the contact radius due to a change in the heat flux can be determined by taking the partial derivative of r_0 with respect to q, Differentiating Eq. (1) with respect to R_1 and R_2 and using the chain rule yields

$$\frac{\partial r_0}{\partial q} = \left(\frac{\kappa}{3}\right)\left[\frac{R_1R_2}{R_1+R_2}\right]^{-2/3}\left[\frac{1}{(R_1+R_2)^2}\right]\left[R_2^2\frac{dR_1}{dq}+R_1^2\frac{dR_2}{dq}\right] \tag{3}$$

where

$$\kappa = [(3\pi/4)F(K_1+K_2)]^{1/3}$$

Each of the terms in Eq. (3) are always positive with the exception of the last bracketed term. Thus, this term determines the sign of $\partial r_0/\partial q$ and strongly influences thermal rectification. The sign of the last bracketed term is dependent on the signs of the two derivatives dR_1/dq and dR_2/dq, The magnitude of these two quantities are complex functions of

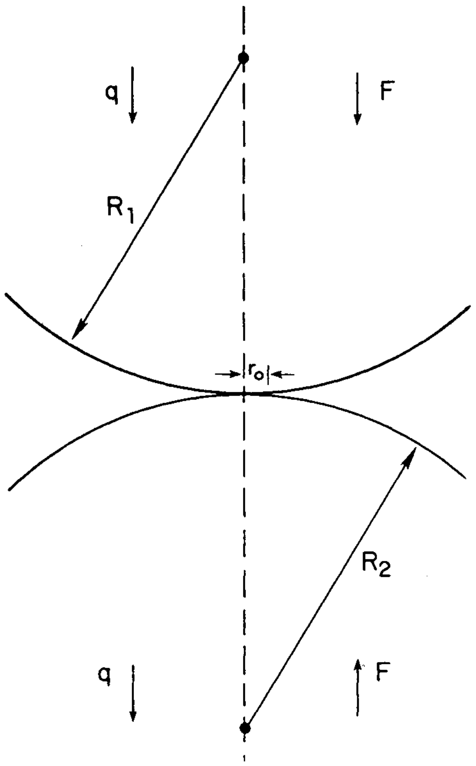

Fig. 1 Contact model between two spheres.

the geometries and thermomechanical properties of the two bodies. [Two studies have indicated that the thermal distortion is proportional to the thermal distortivity, $\alpha(1+\nu)/k$, of the material.] The signs of dR_1/dq and dR_2/dq, however, are readily determined from the direction of the heat flux. For heat flowing into the surface of a sphere, the resulting thermal strains will cause the surface to become more convex, decreasing the radius of the sphere. Hence, dR/dq would be negative. For heat flowing out of the surface of a sphere, dR/dq has a positive value. Thus, dR_1/dq and dR_2/dq will always have opposite signs, and the sign of the bracketed term in Eq. (3) may vary, depending on which of the two spheres has the greater thermal strain and/or which of the radii, R_1 or R_2, is larger.

Existence of Thermal Rectification

Using Eq. (3), thermal rectification can be shown to exist for the contact between two elastic spheres through which heat is flowing. If one arbitrarily assumes $|R_1^2\,(dR_2/dq)| > |R_2^2\,(dR_1/dq)|$, the sign of the bracketed term in Eq. (3) will depend solely on the direction of the heat flux through the junction. Thus, when heat flows from sphere 1 to sphere 2, $R_1^2\,(dR_2/dq)<0$ and $\partial r_0/\partial q<0$; the contact area decreases, increasing the thermal resistance of the junction. For heat flowing from sphere 2 to sphere 1, $R_1^2\,(dR_2/dq)>0$ and $\partial r_0/\partial q>0$; the contact area increases, reducing the thermal resistance at the interface. This dependence of the thermal resistance on the heat flow direction across the interface is thermal rectification.

The results of this analysis have considerable significance. Equation (3) indicates that thermal rectification is a function of the thermal distortions of the contact interfaces which, in turn, are functions of the temperature fields within the contacting bodies. Thus, the geometries of the two contacting bodies, not only the geometries of interfacial surfaces of the bodies, affect the directional bias and the thermal contact resistance. This means that experimental thermal resistance data have applications only for contacts between bodies with overall geometries like those of the test specimens used to generate the data.

Much of the confusion surrounding previous research of thermal rectification has been due to a number of apparent contradictions in the experimental observations. Using Eq. (3), one can explain the existence of these directional bias trends for different parameters qualitatively. The remainder of this Synoptic will address four previously observed experimental trends.

Thermal Rectification Increase due to Increases in the Contact Load

Equation (3) indicates that $\partial r_0/\partial q$ is directly proportional to the load F to the one-third power. Thus, an increase in the contact load will make the thermal rectification of the contact more pronounced since the contact radius change due to heat-flux changes will be greater. In this respect, Eq. (3) differs from earlier theoretical studies that anticipated an opposite trend.

Thermal Rectification Reversal due to Contact Surface Changes

It is possible to explain this phenomenon as a consequence of thermal strain of the contact using the last bracketed term of Eq. (3). If $|R_2^2\,(dR_1/dq)| > |R_1^2\,(dR_2/dq)|$, this term will have the same sign as dR_1/dq. If the opposite is true, the bracketed term will acquire the sign of dR_2/dq. If the two terms are equal, the bracketed term will be zero. It is possible to select values of R_1 and R_2 that will produce each of these three situations, regardless of the relative magnitudes of dR_1/dq and dR_2/dq. Thus, the directional bias of a contact can be reversed simply by altering the surface curvatures of the surfaces. An important aspect of this conclusion is that similar material contacts ($|dR_1/dq| = |dR_2/dq|$) can exhibit thermal rectification and thermal rectification reversal, if they have different surface geometries. The situation in which the bracketed term equals zero is also of interest; in this case, $\partial r_0/\partial q$ equals zero, and thermal rectification does not exist.

Thermal Rectification Reversal due to Heat-Flux Magnitude Changes

It is possible to explain this observation by referring once again to the final bracketed term in Eq. (3). As the heat flux through a junction is increased, the geometries of the contact surface of the two materials, R_1 and R_2, change due to thermal distortion. Thus, the magnitude of the two oppositely signed terms making up the bracketed quantity will change, altering the magnitude, and potentially the direction, of the thermal rectification of the contact.

Let $R_1>R_2$ (no heat flux) and $dR_1/dq=dR_2/dq$. Thus, the last term of Eq. (3) will have the same sign as dR_2/dq. However, for large heat fluxes from material 2 to material 1, R_1 will decrease and R_2 will increase until $R_2>R_1$, whereupon the bracketed term will take on the sign of dR_1/dq, reversing the directional bias.

The Influence of Microscopic Asperity Contacts

The contact model was developed by assuming that the interface between the two adjoining materials was frictionless. Tangential forces produced by friction will oppose the thermal strain movement of the surfaces, resulting in a smaller change in the contact area than would exist for a frictionless junction. This reduction of interfacial movement reduces the thermal rectification of the contact. Also, microscopic asperity contacts cause additional resistance to heat transfer through an interface due to the extra constriction of the heat flow lines. Hence, the apparent thermal rectification of a contact is less if microscopic effects are included, because the change in thermal resistance due to the directional bias is a smaller percentage of the total resistance of the junction.

References

[1]Somers, R. R. II, "The Thermal Contact Conductance of Dissimilar Metals," Ph.D. Dissertation, Univ. of Virginia, Charlottesville, VA, May 1983.